UDC 533:538

INVESTIGATION OF THE STRUCTURE OF ELECTROHYDRODYNAMIC SHOCK WAVES USING THE COMPLETE EQUATION OF MOTION FOR CHARGED COMPONENT INSTEAD OF OHM'S LAW

PMM Vol. 41, № 6, 1977, pp. 1033-1052

V. V. GOGOSOV, V. M. KOROVIN and V. A. POLIANSKII
(Moscow)

(Received September 6, 1976)

Structure of electrohydrodynamic shock waves is investigated using the most general form of Ohm's law, viz. the complete equation of charged particle momenta. It is assumed that outside the wave structure the parameters are defined by the conventional Ohm's law. Additional equations are derived for closing the relationships at the electrohydrodynamic discontinuity. Various mechanisms of surface charge formation at the wave front are investigated. It is shown that the convection term and the ion pressure gradient do not contribute to the formation of surface charge. Allowance in the equation of momenta of charged component of the thermal diffusion term and of the term which defines the effect of the mixture viscous momentum transfer on ion diffusion in the electric field yielded new supplementary relationships at the wave front, which differ from those obtained in [1, 2].

1. Statement of the problem. The structure of electrohydrodynamic shock waves was investigated in [1, 2] with the aim of obtaining a supplementary equation for closing the system of relationships at the discontinuity in the case of absence of surface current at the wave. The model of medium was used there with Ohm's law of the form

$$\mathbf{j}^* = q^* (\mathbf{u}^* + b\mathbf{E}^*) \tag{1.1}$$

where $\mathbf{j}^*$ is the density of electric current, q^* is the density of volume charge (we assume that there is one kind of positively charged particles, ions, so that $q^* > 0$), $\mathbf{u}^*$ is the velocity of medium, b is the coefficient of ion mobility, and $\mathbf{E}^*$ is the vector of electric field intensity.

Equation (1.1) follows from the equation of motion for the ion component

$$\begin{gathered}\frac{\partial}{\partial t} \rho_i^*\mathbf{v}^* + \nabla p_i^* + \operatorname{div} [\rho_i^*\mathbf{v}^*\mathbf{v}^* + \pi_i^* - \rho_i^* (\mathbf{v}^* - \mathbf{u}^*)(\mathbf{v}^* - \mathbf{u}^*)] - \\ q^*\mathbf{E}^* = \mathbf{R}_i^{(1)} \\ \mathbf{R}^{(1)}{}_i = b^{-1} [q^*\mathbf{u}^* - \mathbf{j}^* - q^* D_t \nabla T^* + q^* D_v \mathbf{E}^* W^*] \\ W_{rs} = \frac{1}{2}\left(\frac{\partial u^*{}_r}{\partial x_s} + \frac{\partial u^*{}_s}{\partial x_r}\right) - \frac{1}{3}\delta_{rs} \operatorname{div} \mathbf{u}^* \\ \mathbf{j}^* = q^*\mathbf{v}^*, \ \ p_i{}^* = e^{-1} k q^* T^*\end{gathered} \tag{1.2}$$

where $\rho_i{}^*$, $\mathbf{v}^*$, $p_i{}^*$, and $\pi_i{}^*$ are, respectively, the density, velocity, pressure, and the viscosity stress tensor of the ion component, e is the charge of an ion, T^*

is the temperature, W_{rs}^* are components of the tensor of the mixture strain rate, and k is the Boltzmann constant.

Equation (1.2) was obtained from the Boltzmann kinetic equation for the ion component; the temperature, pressure, and the tensor of viscosity stresses of ions are defined by particle random velocity determined by the mixture mean velocity. Coefficients b, D_t, D_v, and all remaining transport coefficients used below are determined by Grade's method in the thirteen moments approximation, as given in [3].

The shock wave structure is investigated here using Eqs. (1.2) instead of the usual Ohm's law (1.1).

Let us consider the steady flow of medium in an electric field, and select the system of coordinates in which the x-axis is directed downstream. We assume that all quantities depend only on x and that the medium and ion velocities, and the electric field have the components only along the x-axis. In dimensionless form the equations that define such flow with allowance for the medium viscocity and thermal conductivity are

$$\frac{4}{3}\varepsilon\frac{du}{d\zeta} = u + \frac{1}{\gamma M_1^2}\frac{T}{u} - SE^2 - \Pi_* \tag{1.3}$$

$$\frac{\varepsilon}{(\gamma-1)M_1^2 \Pr}\frac{dT}{d\zeta} + \frac{4}{3}\varepsilon u\frac{du}{d\zeta} = \frac{1}{(\gamma-1)M_1^2}T + \frac{1}{2}u^2 + 2SJ(\varphi-\varphi_1) - \Sigma_* \tag{1.4}$$

$$dE/d\zeta = q, \quad d\varphi/d\zeta = -E \tag{1.5}$$

$$\mu\left(T - \frac{\gamma M_1^2}{q^2}\right)\frac{dq}{d\zeta} = -J + q(u+E) + \frac{2}{3}\varepsilon a_v qE\frac{du}{d\zeta} - q(\mu + \varepsilon a_t)\frac{dT}{d\zeta} \tag{1.6}$$

$$p = \rho T, \quad \rho u = 1 \tag{1.7}$$

$$\Pi_* = \Pi - \varepsilon\omega_u, \quad \Sigma_* = \Sigma - \varepsilon\omega_t \tag{1.8}$$

$$\omega_u = \frac{4}{3}\frac{du}{d\zeta}\bigg|_{\zeta=\zeta_1}, \quad \omega_t = \omega_u + \frac{1}{(\gamma-1)M_1^2\Pr}\frac{dT}{d\zeta}\bigg|_{\zeta=\zeta_1}$$

$$\Pi = 1 + \frac{1}{\gamma M_1^2} - SE_1^2, \quad \Sigma = \frac{1}{2} + \frac{1}{(\gamma-1)M_1^2}$$

Equation (1.6) of ion motion has been transformed using the penultimate of Eqs. (1.2) and, tensors of viscous and diffusion stresses of the ion component are neglected.

In these equations the dimensionless parameters are defined by formulas

$$\rho = \frac{\rho^*}{\rho_1^*}, \quad u = \frac{u^*}{u_1^*}, \quad p = \frac{p^*}{p_1^*}, \quad T = \frac{T^*}{T_1^*}, \quad E = \frac{bE^*}{u_1^*}, \quad \varphi = \frac{b\varphi^*}{u^*_1 L} \tag{1.9}$$

$$q = \frac{q^* u_1^*}{|j^*|}, \quad M_1^2 = \frac{\rho_1^* u_1^{*2}}{\gamma p_1^*}, \quad S = \frac{1}{8\pi\rho_1^* b^{*2}}, \quad J = \frac{j^*}{|j_1^*|}, \quad \Pr = \frac{c_p\eta}{\lambda}$$

$$\gamma = \frac{c_p}{c_v}, \quad \zeta = \frac{x}{L}, \quad a_v = \frac{D_v u_1^*}{bl}, \quad a_t = \frac{D_t T^*_1}{u^*_1 l}$$

$$\varepsilon = \frac{l}{L}, \quad \mu = \frac{l_d}{L}$$

and

$$l = \frac{\eta}{\rho_1^* u_1^*}, \quad l_d = \frac{kT_1^* b}{eu_1^*}, \quad L = \frac{u_1^{*2}}{4\pi b |j^*|} \tag{1.10}$$

where p^* and ρ^* are the medium pressure and density, φ^* is the electric potential, c_p and c_v are specific heats, and η and λ are coefficients of viscosity and thermal conductivity of the medium. In this case the equation of continuity for ions implies that $j^* = \text{const}$. Quantities Π and Σ in Eqs. (1.3) and (1.4) are constants of integration determined by flow parameters at some point $\zeta = \zeta_1$, and at that point are denoted by subscript 1. Below we consider the flow of a weakly ionized gas $(\rho_i^* / \rho^* \ll 1)$.

Using the results obtained in [3], for a_v and a_t we have in this case

$$a_v = \frac{6\gamma b_1^* c_1 M_2{}^1}{c_1^*}\left(1 + \frac{0.3A_{aa}^* Q_{aa}}{c_1 Q_{ia}}\right), \quad a_t = \frac{72 b_1^*}{f_1^*}\left(A_{ia}^* - A_{aa}^* \frac{Q_{aa}}{Q_{ia}}\right) \tag{1.11}$$

$$b_1^* = 0.25\,(1.2C_{ia}^* - 1), \quad c_1 = 0.25\,(1 - 0.6A_{ia}^*), \quad c_1^* = 1 + 0.6A_{ia}^*$$

$$f_1^* = 5.5 + 1.6A_{ia}^* - 1.2B_{ia}^*$$

where Q_{aa} and Q_{ia} are transport cross sections of elastic collisions of neutral particles between themselves and of charged particles with neutral ones, respectively. Formulas for $Q_{\alpha\beta}$ and for $A_{\alpha\beta}^*$, $B_{\alpha\beta}^*$, and $C_{\alpha\beta}^*$ appear in [3]. When particles interact as elastic solid spheres, the last three quantities are equal unity. These quantities were determined in [4, 5] for other interaction laws and various gas mixtures (*) For molecules with a Maxwellian interaction potential $C_{ia}^* = {}^5/_6$ so that $a_t = a_v = 0$.

We shall solve the problem of the electrohydrodynamic shock wave structure as formulated in [2, 6, 7]. It follows from the first of Eqs. (1.5) that the flow is everywhere nonuniform when $q \neq 0$. It is assumed that a zone $\Gamma\{\zeta_2 \leqslant \zeta \leqslant \zeta_3\}$ in which the flow parameter gradients are considerable in comparison with those outside it, exists in the flow region. The variation of parameters in the zone of considerable gradients is called the structure of shock wave. In the considered one-dimensional problem it is possible to use the characteristic quantities for making up several conbinations (1.10) of dimension of length It was shown in [2, 6] in the investigation of the wave structure determined by viscosity and thermal conductivity of the medium that the characteristic dimension of the structure zone is of the order of length l, while the characteristic dimension of flow parameter nonuniformity is of the order of length L. Let us clarify the physical meaning of length l_d. In the case of weakly ionized gas, using the formula given in [3] for the mobility coefficient, we can write

*) See also the paper by Iu. N. Beliaev, V. A. Polianskii, K. V. Romashina, and E. G. Shapiro, Transport phenomena in gases and gas mixtures, pt. 1. Collision Integrals. Report of the Institute of Mechanics of MGU, No. 1802, 1976.

$$l_d = \frac{kT_1^*}{m} \frac{\tau_{ia}}{u_1^*} = \frac{v_t^*}{u_1^*} l_i \sim l_i, \quad v_t^* = \left(\frac{kT_1^*}{m}\right)^{1/2}, \quad l_i = v_t^* \tau_{ia} \tag{1.12}$$

where m is the mass of an ion, τ_{ia} is the effective time of mean free path run of ions, and v_t^* is the ion thermal velocity. It is seen from (1.12) that the order of magnitude of l_d is the same as that of charged particles mean free path l_i.

The ratio of lengths l_d / l (inverse of the Schmidt number) depends on the cross sections of particle collisions and in this case is

$$\frac{l_d}{l} = \frac{\mu}{\varepsilon} = \frac{0.9A^*_{aa}Q_{aa}}{Q_{ia}}, \quad \delta = \frac{\mu}{\varepsilon} \tag{1.13}$$

For many gas mixtures this ratio is small owing to the considerable difference between collision cross sections.

We assume that parameters $\varepsilon \ll 1$ and $\mu \ll 1$, and select points ζ_1 and ζ_4 outside the Γ-zone close to points ζ_2 and ζ_3, respectively. Equations (1.3), (1.4), and (1.6) show that in the flow regions $\zeta \ll \zeta_1$ and $\zeta \gg \zeta_4$, where the derivatives of velocity, temperature, and charge density are of order unity, the macroscopic parameters are linked by formula (1.1) to within quantities of order ε and μ, and by integrals of perfect flow which obtain from (1.3) and (1.4) by equating the right-hand sides of these equations to zero and setting $\Pi_* = \Pi$ and $\Sigma_* = \Sigma$. The problem of electrohydrodynamic wave structure consists of determining the integral curves of system (1.3)-(1.7) which connect regions $\zeta \ll \zeta_1$ and $\zeta \gg \zeta_4$ of perfect flow.

The qualitative analysis of behavior of such integral curves is presented below with successive calculation of terms in Eq. (1.6). The case in which charged particles move downstream $(j^* > 0)$ is considered, hence $J = 1$. It is assumed that the parameter of electrohydrodynamic interaction is small $(S \ll 1)$, hence the effect of the electric field on the motion of mixture is small and the Schmidt number $\delta^{-1} \gg 1$. We set the Prandtl number $\Pr = 0.75$. Then, as shown in [8], Eq. (1.4) can be integrated, and instead of (1.3), (1.4), and (1.6) we obtain for the problem of wave structure for small S the equations

$$T = 1 + \frac{(\gamma-1)M_1^2}{2}(1-u^2), \quad \varepsilon\frac{du}{d\zeta} = L_2, \tag{1.14}$$

$$L_2 = \frac{3(\gamma+1)(u-1)(u-\beta)}{8\gamma u}$$

$$\beta = (\gamma - 1 + 2M_1^{-2})(\gamma-1)^{-1}$$

$$\mu\frac{dq}{d\zeta} = \frac{L_1}{L_3}, \quad L_1 = -1 + q(u+E) + \frac{2}{3}a_v qEL_2 + (\gamma-1)M_1^2 qu(a_t+\delta)L_2, \quad L_3 = 1 + 0.5(\gamma-1)M_1^2(1-u^2) - \gamma M_1^2 q^{-2} \tag{1.15}$$

The equality $L_2 = 0$ obviously follows from the integrals of perfect flow.

The second of Eqs. (1.14) and the first of Eqs. (1.15) and of Eqs. (1.5) constitute a closed system of equations for u, E, and q. Since the independent variable does not explicitly appear in the right-hand sides of these equations, it is convenient to analyze the behavior of integral curves if equations in the uEq-space.

Eliminating in equations the variable ζ, we obtain

$$dq / du = f_1,\ f_1 = L_1 / \delta L_2 L_3 \quad (1.16)$$

$$dE / du = f_2,\ f_2 = \varepsilon q / L_2 \quad (1.17)$$

$$dE / dq = \mu q L_3 / L_1 \quad (1.18)$$

Since by assumption $q^* > 0$ and $u^* > 0$, hence the integral curves of the systems of Eqs. (1.16)-(1.18), which have a physical meaning lie in the region $q > 0$ and $u > 0$ of the uEq-space, and from the solution of the problem of gasdynamic shock wave structure for small S follows that the velocity of gas can vary within the limits $\beta \leqslant u \leqslant 1$.

The friction force $\mathbf{R}_i^{(1)}$ between the ion and neutral components, which appears in Eq. (1.2) of ion motion, was determined by Grade's method in linear approximation with respect to streams [3]. The linear approximation is valid when the characteristic lengths of macroscopic parameter variation exceed the mean free path of particles. Such conditions may not be satisfied in the problem of shock wave structure. It should be noted, however, that the analysis of the structure of conventional gas-dynamic shock waves by means of the Navier—Stokes equations yields for low intensity shock waves the results that are in a fair agreement with those obtained by methods of kinetic theory. Hence it is possible to consider the use of Eqs. (1.2) for determining the processes taking place inside the wave structure to be justified, at least for low intensity shock waves.

2. Shock wave structure with allowance in the equation of motion of the charged component for the ion pressure gradient and for the convection term. Let us investigate the shock wave structure in a gas of Maxwellian molecules $(a_v = a_t = 0)$. Since the quantity $|L_2| \lesssim 1$ lies in that region of the uEq-space where lie the integral curves of Eqs. (1.16)—(1.18) that have a physical meaning, it is possible to disregard in the formula for L_1 the term with δL_2 as being smaller than unity.

We construct in the uEq-space the planes $u = 1$ and $u = \beta$ (Fig. 1) and denote these by L_2°. We draw in these planes hyperbolas Γ_i° $(i = 1, 2, 3, 4)$ whose equations are

$$q = \pm (1 + E)^{-1}, \quad q = \pm (\beta + E)^{-1}$$

and whose asymptotes $E = -1$ and $E = -\beta$ are vertical and lie in the plane $E = -u$. Hyperbolas Γ_1° and Γ_2° are in that part of the uEq-space in regions of physical space outside zone Γ of large gradients in which the electric current $j^* > 0$ and hyperbolas Γ_3° and Γ_4° lie in that part where $j^* < 0$ $(J = -1)$.

In the space comprised between the planes $u = 1$ and $u = \beta$ we construct surface L_1° defined by the equation $L_1 \equiv -1 + q(u + E) = 0$. That surface intersects the planes $u = 1$ and $u = \beta$ along hyperbolas Γ_i°. The intersection of surface L_1° with planes $E = \text{const}$ is along hyperbolas l_1° which are defined by the equations $q = (u + E)^{-1}$ with $E = \text{const}$. When $E > -\beta$, the vertical asymptotes lie in region $u < \beta$, while for $-1 \leqslant E \leqslant -\beta$ they are in region $\beta \leqslant u \leqslant 1$. It will be seen that all these asymptotes are in the plane $u = -E$ which separates surface L_1 in two parts that correspond, respectively, to the positive $(u + E > 0)$ and negative $(u + E < 0)$ electric current in regions of the physical space outside zone Γ. Above surface L_1° we have $L_1 > 0$, and below it $L_1 < 0$. Equations (1.16) and (1.17) show that the integral curves intersect surface L_1° in the planes $q = \text{const}$. The plane $u = \beta^{1/2}$ divides the considered region into parts that correspond, respectilvely, to supersonic $(u > \beta^{1/2})$ and subsonic flows. The relative position of surface L_1° and the cylindrical surface L_3° .(the latter defined by the equation $L_3 = 0$) whose gereratrix is parallel to the E -axis can vary depending on the number M_1.

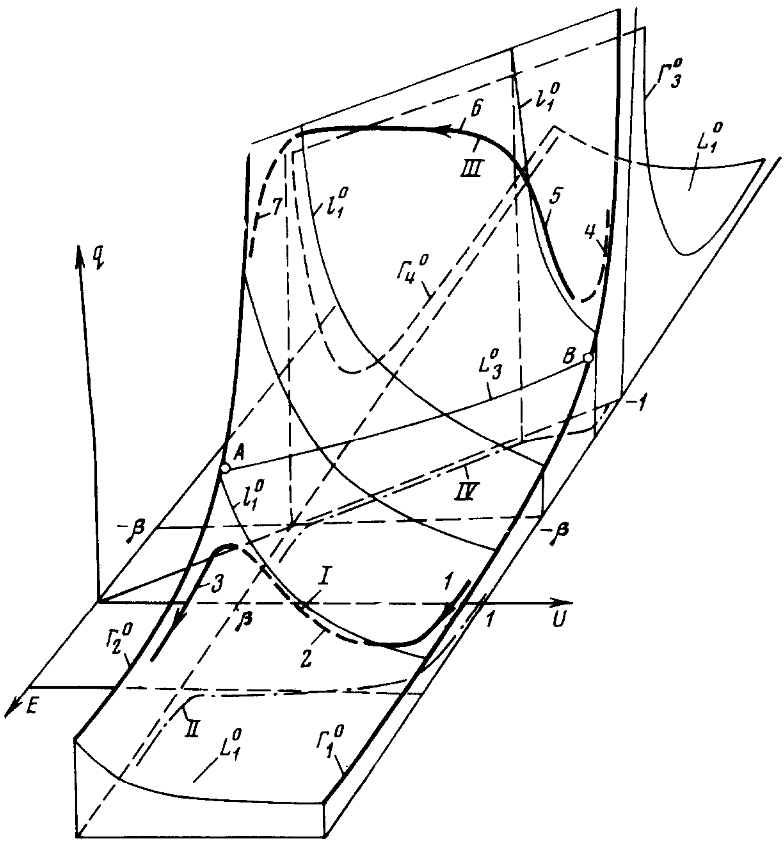

Fig. 1

We denote by l_3° the line of intersection between surface L_3° and some plane $E = E_1 = \text{const}$. Surfaces L_3° and L_1° intersect in region $\beta \leqslant u \leqslant 1$ along the line AB (Fig. 1). The coordinates of points A and B are defined by

$$u_A = \beta, \quad E_A = q_A^{-1} - \beta, \quad q_A = \gamma^{1/2} M_1 \, [1 + {}^1/_2(\gamma - 1) \times M_1^2 \, (1 - \beta^2)]^{-1/2} \tag{2.1}$$

$$u_B = 1, \quad E_B = -1 + \gamma^{-1/2} M_1^{-1}, \quad q_B = \gamma^{1/2} M_1$$

At the part of surface L_1° that lies below the line AB, the quantity $L_3 < 0$, while at points L_1° above line AB the quantity $L_3 > 0$. If $E_1 > E_A$ or $E_1 < E_B$, lines l_1° and l_3° do not intersect in region $\beta < u < 1$, and in the first case l_1° lies in the region where $L_3 < 0$, while in the second $L_3 > 0$ at points l_1°. At points which do not lies on line AB the integral curves intersect surface L_3° where the tangent vector is directed along the q -axis.

Let $ds^2 = du^2 + dE^2 + dq^2$ be the square of an arc element of the integral curve. The directional cosines of the vector $\tau\ (\alpha_1, \alpha_2, \alpha_3)$ tangent to the integral curve are of the form

$$\alpha_1 = du/ds = \pm (1 + f_1^2 + f_2^2)^{-1/2}, \quad \alpha_2 = dE/ds = \alpha_1 f_2 \qquad (2.2)$$
$$\alpha_3 = dq \,/\, ds = \alpha_1 f_1$$

The sign of α_1 depends on the direction of motion along the integral curve. We shall consider positive the motion (with $\alpha_1 > 0$) in the direction of increasing u. To obtain a fuller picture of the integral curve behavior we shall, also, analyze the order of integral curves with respect to parameters ε and μ of torsion χ using formulas [9]

$$\chi = \Delta_1^{-1}\Delta_2, \quad \Delta_1 = (u^{(2)}q^{(1)} - u^{(1)}q^{(2)})^2 + (E^{(2)}u^{(1)} - E^{(1)}u^{(2)})^2 + \qquad (2.3)$$
$$(E^{(2)}q^{(1)} - E^{(1)}q^{(2)})^2, \quad A^{(n)} = d^nA \,/\, d\xi^n$$

The quantity Δ_2 is a third order determinant whose rows are of the form $(u^{(n)}, E^{(n)}, q^{(n)})$, $n = 1, 2, 3$. In the region where $q < \varepsilon^{-1}$ and L_1 is within the limits $\delta < | L_1 | < \varepsilon^{-1}$ we obtain from (2.3) the estimate

$$\chi \sim \varepsilon \mid L_2 \mid (\varepsilon^2 q^2 + L_2^2)^{-1} \qquad (2.4)$$

If $L_1 = O\,(\mu)$, from (2.3) we have

$$\chi \sim \varepsilon \mid L_2 \mid (\varepsilon^2 q^2 + L_2^2)^{-1} (\varepsilon q + \mid L_2 \mid)^{-1} \qquad (2.5)$$

It will be seen that the sections of integral curves which lie in regions where $q < \varepsilon^{-1}$ and $L_2 = O\,(1)$ are flat, since there $\chi \sim \varepsilon$.

Let us establish the direction of vector τ at an arbitrary point away from surface L_1°. When $q \ll \varepsilon^{-1}$ and $| L_2 | \gg \varepsilon$, we have $f_1 = O\,(\delta^{-1})$ and $f_2 \gg \varepsilon$. Formulas (2.2) show that $\alpha_1 = O\,(\delta)$, $\alpha_2 = O\,(\varepsilon\delta)$,. and $\alpha_3 = O\,(1)$ and the tangent vector is directed along the q -axis, hence away from surface L_1° the slope of integral curves is close to vertical. At points of surface L_1° away from L_2° we have $\alpha_1 = O\,(1)$, $\alpha_2 = O\,(\varepsilon)$, and $\alpha_3 = 0$ and the integral curves intersect surface L_1° where the tangent vector is directed along the u -axis.

Let us consider the behavior of integral curves in region $\beta < u < 1$, $E + \beta \gg \varepsilon$. It follows from (2.2) that away from surface L_2° the quantity $f_2 = O\,(\varepsilon)$ when $q \ll \varepsilon^{-1}$, hence, along the integral curves $dE \,/\, ds = O\,(\varepsilon)$ the electric field varies slightly in the indicated regions, and each integral curve lies in a small neighborhood of the related plane E = const.

Let us determine in region $\beta < u < 1$ the projection of the field of integral curves onto any plane $E = E_1$ (Fig. 2). Above surface L_1° under surface L_3°, the quantity $f_1 > 0$. The integral curve slope relative to the u -axis is positive. The slope changes its sign at points of surface L_3°.

Below surface L_1°, the quantity $f_1 < 0$ and the slope of integral curves relative to the u.-axis is negative. As already indicated, the slope of integral curves away from surface L_1° is close to vertical when δ is small.

Let us consider the integral curve passing through an arbitrary point a lying below and away from curve l_1° (type II integral curve). At that point its slope relative to the u -axis is negative and close to vertical. The integral curve tends to approach curve l_1° from below, but cannot intersect it since an intersection is only possible at zero slope, for which it is necessary that f_1 changes its sign in the region below l_1°. Because of this the integral curve turns and runs in the small neighborhood along curve l_1° with a finite negative slope.

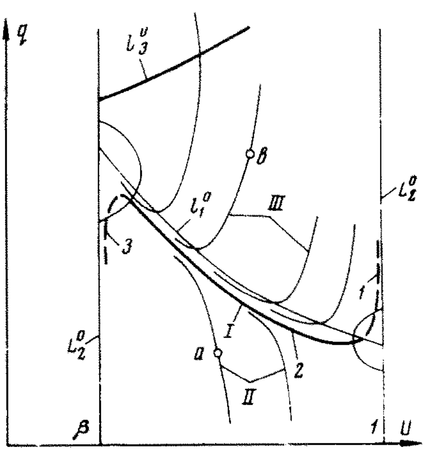

Fig. 2

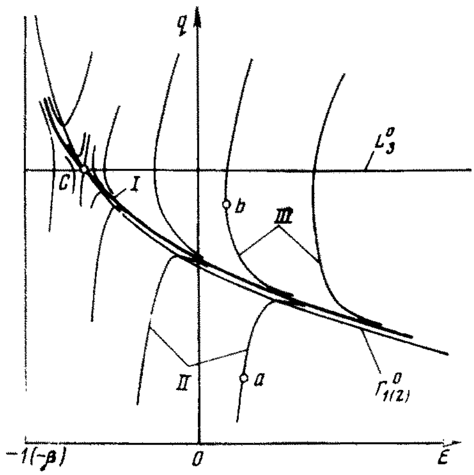

Fig. 3

An integral curve of type III which passes through point b lying above and away from curve l_1° has at that point a positive slope relative to the u -axis and is close to vertical. Integral curves of type III , unlike the type II curves, intersect curve l_1° with zero slope beyond which the slope becomes negative and the integral curve runs in the small neighborhood of l_1° along it. Since a single integral curve of Eqs. (1.16) and (1.17) passes through each point, there exists in the considered region an integral curve (a separatrix) that passes in the δ- neighborhood of l_1° and separates these two sets of integral curves (curves I in Figs. 1 and 2). The separatrix and integral curves lying close to it join the neighborhood of lines $u = 1$ and $u = \beta$, shown in Fig. 2 by arcs of circles.

Let us consider the small neighborhoods of planes $u = 1$ and $u = \beta$ in region $\beta < u < 1$. If $|L_1| \gtrsim \mu$ and $L_2 = O(\varepsilon^3)$, then, as seen from (2.2), $du / ds \lesssim \varepsilon^2$. Estimates (2.4) and (2.5) show that then $\chi \lesssim \varepsilon$. Hence in the small neighborhoods of planes $u = 1$ and $u = \beta$ lie flat sections of integral curves that run along these planes. To establish the qualitative pattern of behavior of such flat integral curves we, first, determine the projections of these onto the planes $u = 1$ and $u = \beta$ (Fig. 3). These integral curves are defined by equations derived from (1.18) by the successive substitution in the expressions for L_1 and L_3 of $u = 1$ and $u = \beta$. The hyperbolas $\Gamma_{1,2}^\circ$ are evidently zero-slope isoclines, while the lines of intersection of surface L_3° with planes $u = 1$ and $u = \beta$ are vertical isoclines of the integral curves of these equations. When $q \ll \varepsilon^{-1}$, the slope of integral curves away from line $\Gamma^{\circ}_{1(2)}$ is close to vertical, since $\mu \ll 1$. In the μ-neighborhood above hyperbola $\Gamma^{\circ}_{1(2)}$ there are two sets of integral curves divided by a separatrix. Integral curves of one set (II) run along hyperbola $\Gamma^{\circ}_{1(2)}$ with decreasing E, intersect it and go down; curves of the second set (III) do not intersect $\Gamma^{\circ}_{1(2)}$ and go upward. The separatrix which runs along hyperbola $\Gamma^{\circ}_{1(2)}$ in the μ-neighborhood above it is shown in Fig. 3 by the heavy line (I). Point c at which $L_3 = 0$ and $L_1 = 0$ is a singular point of Eq. (1.18). Analysis shows that it is of the saddle point kind, and that the separatrix I is one of the singular solutions. To the left of point c the separatrix lies below hyperbola $\Gamma^{\circ}_{1(2)}$.

The integral curves passing through points lying in the μ-neighborhood of lines $\Gamma^{\circ}_{1(2)}$ above the surface L_1° in region $\beta < u < 1$ and close to planes L_2° in their ε^3-neighborhood behave similarly, forming in that region a bundle around $\Gamma^{\circ}_{1(2)}$ from which branch out integral curves which run into region $\beta < u < 1$.

The direction of motion is indicated in figures by arrows. These directions correspond in the physical plane to the downstream motion. They are determined by the first of Eqs. (1.5) which implies that when $q > 0$ the electric field always increases downstream, and from the second of Eqs. (1.14) follows that velocity in the region $\beta < u < 1$ decreases downstream.

Let us consider one of the integral curves that branch from the plane $u = 1$ into region $\beta < u < 1$ at a certain intensity $E_1 > -\beta$ of the electric field (curve I in Fig. 4). It follows from (2.5) that with increasing L_2 the quantity χ first rapidly increases ($\chi \sim \varepsilon^{-1}$ when $|L_2| \sim \varepsilon$) and then decreases (when $|L_2| > \varepsilon$, $\chi \sim \varepsilon$). In accordance with what was said above this means that the integral curve passes from the plane $u = \text{const}$ to the plane $E = \text{const}$, and that such passage is accompanied by a fairly small variation of u. Let us consider the pattern of parameter variation along the integral curve. It follows from (2.2) that when the motion is in the direction of decreasing u ($\alpha_1 < 0$) the quantity q must decrease in the region of $L_1 > 0$ (above L_1°), $L_2 < 0$ (to the left of $u = 1$), and $L_3 < 0$ (below L_3°). Since in the motion along surface L_1° in the plane $E = \text{const}$ in the direction of decreasing u the quantity q increases, hence the integral curve must intersect surface L_1° at some point close to the plane $u = 1$.

Behavior of the integral curve in the plane $E = \text{const}$ was considered above, and it was shown that it runs in the δ-neighborhood of line l_1° in the region where $L_1 < 0$ (section 2 of curve I in Figs. 2 and 4). In approaching the plane $u = \beta$, L_2 decreases and χ begins to increase. Formula (2.4) shows that when $|L_2| \sim \varepsilon$, the twist χ is of order unity. and the integral curve deviates from the plane $E = \text{const}$ in the direction of increasing E, as can be seen from the second of Eqs. (2.2). With further approach to the plane $u = \beta$, the quantity χ becomes again small, and the curve passes from the neighborhood of the plane $E = \text{const}$ to that of the plane $u = \text{const}$. However motion in the direction of considerable E (and smaller u and $\alpha_1 < 0$) under surface L_1° is impossible, since the derivative $dq / ds > 0$ when $L_1 < 0$, and when $u = \text{const}$ the quantity q decreases on surface L_1° when E increases. The integral curve must, consequently, intersect surface L_1° near the plane $u = \beta$. After that it runs in the ε^3- neighborhood of that plane in the μ-neighborhood of line Γ_2° and along it. (section 3 of curve I).

We denote by $\Delta\zeta_u$, $\Delta\zeta_E$ and $\Delta\zeta_q$ the characteristic distances in which the velocity, the field, and the charge density, respectively, vary by virtue of the second of Eqs. (1.14), the first of Eqs. (1.5), and the first of Eqs. (1.15) by an order of unity. In sections 1 and 3 of the integral curve I near lines $\Gamma_{1,2}^\circ$ the lengths $\Delta\zeta_u \sim \varepsilon^{-2}$, $\Delta\zeta_E \sim 1$, $\Delta\zeta_q \sim 1$, since there $|L_2| \sim \varepsilon^3$ and $L_1 \sim \mu$.

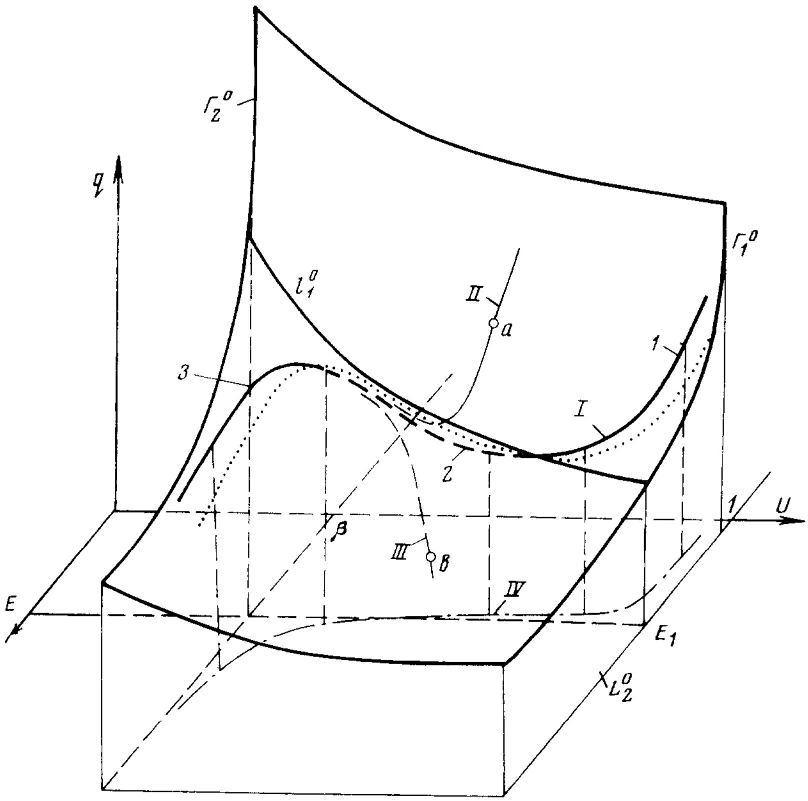

Fig. 4

In the physical plane these sections correspond to a supersonic (1) and a subsonic (3) flow at constant velocity, where the contribution of the pressure gradient and of the convection term in Ohm's law is negligibly small, and the variation of the field and density of charge are related by formulas $q = (u + E)^{-1}$, $u = 1$, and $u = \beta$ to within smalls of order μ. In section 2 which runs in the δ-neighborhood of line l_1° the lengths $\Delta\zeta_u \sim \varepsilon$, $\Delta\zeta_q \sim \varepsilon$, $\Delta\zeta_E \sim 1$, since there $|L_1| \sim \delta$ and $|L_2| \sim 1$. In the physical ε-plane that section corresponds to a flow region of order ε in which the charge density and the velocity (and consequently the gas temperature and density) vary considerably, while the variation of the electric field is small. This section specifies the flow structure in region Γ of considerable gradients, since there the derivatives $u' \sim \varepsilon^{-1}$ and $q' < \varepsilon^{-1}$. Thus the integral curve I connects regions $\zeta \ll \zeta_1$ and $\zeta \gg \xi_4$ of perfect flow, and defines the structure of a shock wave along which the electric field is constant

$$E = E_1 = \text{const} \tag{2.6}$$

So far we considered the case when line l_1° in whose δ - neighborhood passes the integral curve, does not intersect in region $\beta < u < 1$ line l_3° of the intersection of surface L_3° with the plane $E = E_1$. The condition for the electric field intensity E_1 ahead of the wave front, when the opposite case is realized, is of the form $E_B < E_1 < E_A$. We denote by u_* and q_* the velocity and charge density at the point of intersection of curves l_1° and l_3° with the corresponding electric field intensity E_1. Then $q_* \ll \varepsilon^{-1}$ and, when $|L_2(u_*)| \gg \varepsilon$, then in the neighborhood of the intersection point (u_*, E_1, q_*) the field of integral curves is flat. The point (u_*, E_1, q_*) is singular and of the kind of focus of Eq. (1.16) (Fig. 5).

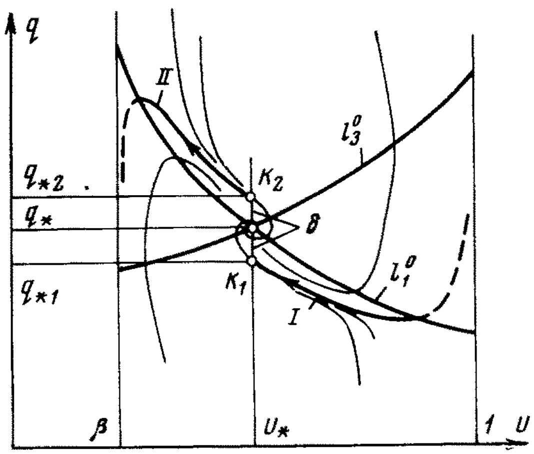

Fig. 5

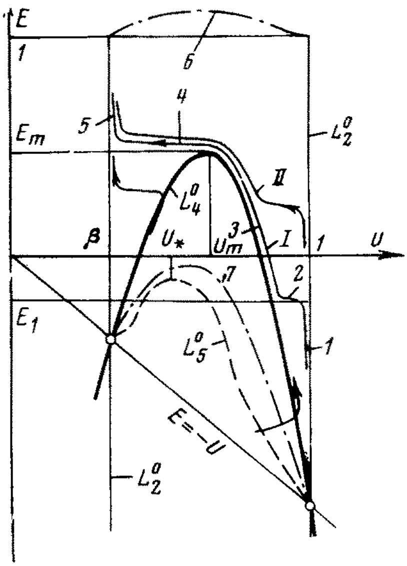

Fig. 6

The method described above can be used for showing that in region $u < u_*$ there exists above line l_1° the separatrix II which separates the integral curves running upward from those that intersect line l_1° and run downward. Integral curve II in approaching the plane $u = \beta$ diverges from the plane $E = E_1$ and moves toward considerable E, it intersects surface L_1° and runs under it in the μ -neighborhood of line Γ_2° in the direction of increasing E. In region $u > u_*$ the integral curve I is a separatrix. Both separatrices curl around the singular point. Thus, when $E_B < E_1 < E_A$, no continuous line connecting regions of supersonic and subsonic perfect flows exists. It is however possible to derive a solution defining the wave structure by inserting inside it a jump of electric charge density from point $k_1(u_*, E_1, q_{*1})$ of the integral curve I to point $k_2(u_*, E_1, q_{*2})$ of the integral curve II at constant values of velocity and electric field. The structure of such jump can be obtained by introducing in the analysis one more dissipation mechanism, viz. the viscosity of charged particles. In the case of small δ considered here intensity of that shock is small.

Let us now consider the case when the initial intensity of the field ahead of the wave front satisfies the conditions $-\beta > E_1 > -1$ and $E_1 < E_\beta$ (curves l_1° and l_3° do not intersect in region $\beta < u < 1$). Let us investigate the behavior of integral curve III (Fig. 1) which runs in the μ-neighborhood of line Γ_1° under the surface L_1° (since in this case $L_3 > 0$) and moves away from the plane $u = 1$ when the initial field intensity is $E = E_1$. That curve intersects surface L_1° in the ε-neighborhood of the plane $u = 1$ with its tangent vector in the plane $q = \text{const}$, then runs along line l_1° in its δ-neighborhood above surface L_1° in the direction of decreasing u. Within the considered range of field intensities $(-1 < E_1 < -\beta)$ the vertical asymptote of line l_1°, which lies in the plane $u + E = 0$, is inside region $\beta < u < 1$. Because of this lines l_1° and Γ_2° do not intersect and, when $E_1 < -\beta$, the integral curve III which runs along l_1° cannot approach line Γ_2° so as to bring the order of magnitude of L_2 to that of ε. But, when the integral curve runs along l_1° it approaches the plane $u + E = 0$ and then the quantity q evidently increases.

Let us investigate the direction of the vector tangent to the integral curve when the order of magnitude of q becomes ε^{-1}. From (1.16) and (1.17) we have $f_1 = O(1)$ and $f_2 = O(1)$. Consequently all α_i are of the same order, since (2.2) implies that $\alpha_1 = O(1)$. Estimate (2.4) shows that the twist χ remains small. The integral curve when passing along l_1° approaches the plane $u + E = 0$ and gradually diverges from the plane $E = E_1$ in the direction of increasing E (Fig. 1). The points of surface L_1° at which $q \sim \varepsilon^{-1}$ lie in the ε-neighborhood of the plane $u + E = 0$. Hence the integral curve which for these values of q runs along surface L_1° runs by the same token along the plane $u + E = 0$ and can approach the plane $u = \beta$ so as to bring the order of L_2 to ε. When $|L_2| \sim \varepsilon$ and $q \sim \varepsilon^{-1}$, components α_2 and α_3 of the tangent vector are of the same order, and $\alpha_1 = O(\varepsilon)$ is a tangent vector collinear with the plane $u = \beta$. The integral curve intersects surface L_1° near the intersection line of planes $u + E = 0$ and $u = \beta$, and then runs along the plane $u = \beta$ under surface L_1° close to line Γ_2° in the direction of increasing E.

It can be shown that, as previously, sections 4 and 7 of the integral curve III correspond in the physical plane to supersonic and subsonic flows subjected to Ohm's law of the form (1.1). Section 5 where $q \sim 1$ and $L_1 \sim \delta$ corresponds in the physical plane to a narrow region of considerable variation of u and q and other gasdynamic parameters with small variation of the electric field. In section 6 where $q \sim \varepsilon^{-1}$ and $L_1 \sim \delta$ we have $\Delta\zeta_u \sim \varepsilon$, $\Delta\zeta_E \sim \varepsilon$, $\Delta\zeta_q \sim \varepsilon$. Consequently this section corresponds to a narrow region of abrupt change of all flow parameters. Projection of the integral curve III onto the plane $q = 0$ is shown in Fig. 1 by the dash-dot line (curve IV).

Thus, when the initial electric field intensity ahead of the wave front satisfies the condition

$$-1 < E_1 < E_B \tag{2.7}$$

it is possible to construct a continuous integral curve which connects regions $\zeta \leqslant \zeta_1$ of supersonic and $\zeta \geqslant \zeta_4$ subsonic perfect flows and defines the shock wave structure in a varying electric field. The field intensity E_2 behind the wave front is related to velocity by the expression

$$u_2 + E_2 = 0 \tag{2.8}$$

When the initial field intensity is

$$E_B < E_1 < -\beta \tag{2.9}$$

it is not possible to use the model considered here for constructing a continuous structure, since then there is inside region $\beta < u < 1$ a point of intersection of curves l_1° and l_3° which for finite q represents a kind of focal point of integral curves that pass there close to the plane $E = E_1 = \text{const}$. The position here is similar to that considered above. It is possible to determine the structure by introducing in the neighborhood of the singular point a shock of q at constant velocity a and electric field intensity. Such jump can be smoothed by complicating the model of medium by the introduction of viscosity of charged particles.

The allowance for the gradient of charged particle pressure in Ohm's law yields for the electric field at the wave front the same relationships and conditions as those that are valid in the case of application of the conventional Ohm's law (1.1) [2]. Allowance for the convection term makes it impossible in some cases to obtain a continuous structure, although the relationships at the wave front remain unchanged.

3. Shock wave structure with allowance for the thermal diffusion term in Ohm's law. Relationships of a new kind for the electric field at the wave front are obtained by allowing in the formula for L_1 (second of formulas (1.15)) for thermal diffusion terms ($a_t \neq 0$). First, we set $\mu = 0$ and $a_v = 0$. The input system of equations then reduces to the single equation

$$\frac{dE}{du} = \frac{\varepsilon}{L_2 L_4}, \quad L_4 = E + u + \alpha (u - 1)(u - \beta) \tag{3.1}$$

$$\alpha = 3 \ (\gamma^2 - 1) \ M_1^2 a_t / 8\gamma$$

We shall analyze Eq. (3.1) in the uE-plane. We construct in region $u > 0$ the lines L_2° and L_4° whose equations are, respectively, $L_2 = 0$ and $L_4 = 0$. These lines are indicatrices of vertical directions of integral curves of Eq. (3.1). Line L_4° is a parabola which passes through the intersection points of lines $u = 1$ and $u = \beta$ with line $u + E = 0$. The coordinates of the parabola vertex are

$$u_m = \frac{\gamma}{\gamma+1}\left(\frac{1}{\gamma M_1^2} + 1\right) - \frac{4\gamma}{3(\gamma^2-1)a_t M_1^2} \tag{3.2}$$

$$E_m = \frac{3(\gamma-1)a_t(M_1^2-1)^2}{8\gamma(\gamma+1)M_1^2} - \frac{\gamma}{\gamma+1}\left(1 + \frac{1}{\gamma M_1^2}\right) + \frac{2\gamma}{3(\gamma^2-1)a_t M_1^2}$$

If the Mach number M_1 ahead of the wave front satisfies the condition

$$M_1^2 > 1 + \frac{4\gamma}{3(\gamma-1)a_t} \tag{3.3}$$

the parabola vertex lies to the right of line $u = \beta$.

The qualitative pattern of behavior of integral curves of Eq. (3.1) is shown in

Fig. 6 for the case when $\alpha > 0$ and the inequality (3.3) is satisfied.

Let us consider the integral curve I (Fig. 6) which passes near the initial point $(1, E_1)$ when $E_1 < E_m$. Section 1 of that curve which passes in the ε-neighborhood of line $u = 1$, to which in the physical plane corresponds a perfect supersonic flow at constant speed with variables E and q satisfying Ohm's law of the form (1.1). In the ε-neighborhood of point $(1, E_1)$ integral curve I deviates from line $u = 1$ and runs with a small negative slope in the direction of decreasing u (section 2). When approaching line L_4^c the curve turns and then runs along that line in its ε-neighborhood in the direction of increasing E (section 3), because it cannot intersect line L_4° with a vertical slope without changing the sign of slope. Having reached the ε-neighborhood of the parabola vertex (u_m, E_m), the integral curve then runs with a small negative slope and approaches line $u = \beta$ (section 4). Then the slope increases in the ε-neighborhood of point (β, E_m) and the curve runs along line $u = \beta$ in its ε-neighborhood (section 5). It follows from the equation $dE / d\zeta = L_4^{-1}$ and the second of Eqs. (1.14) that in sections 2 and 4 the characteristic lengths $\Delta\zeta_u \sim \varepsilon$, and $\Delta\zeta_E \sim 1$, since there $|L_2| \sim 1$ and $L_4 \sim 1$, and in section 3 the lengths $\Delta\zeta_u < \varepsilon$ and $\Delta\zeta_E \sim \varepsilon$, since there $|L_2| \sim 1$ and $L_4 \sim \varepsilon$. Hence in the physical plane narrow flow regions with considerable velocity variations and a nearly constant electric field of intensities E_1 and E_m correspond, respectively, to sections 2 and 4. Section 3 corresponds to a narrow region of considerable variation of velocity and electric field with intensity E changing from close to E_1 to close to E_m. Section 5 corresponds to a perfect subsonic flow.

The integral curve I, thus, describes the variation of velocity and electric field in the shock wave structure at whose front a surface charge is produced by the thermal diffusion process. The field intensity behind the wave is related to parameters of flow ahead of the wave front by the formula

$$E_2 = E_m \tag{3.4}$$

where E_m is defined by the second of formulas (3.2). The surface charge intensity is $\sigma = (E_m - E_1)\, u_1^*/4\pi b$.

It will be seen from Fig. 6 that the surface charge at the wave front can be produced by thermal diffusion only when the Mach number ahead of the wave satisfies condition 3 and the initial field intensity $E_1 < E_m$. The second of formulas (3.2) implies that $E_m > 0$ when $M_1 > M_*$, where M_* is the greatest root of the equation $E_m = 0$ which is biquadratic with respect to M_1, and M_* satisfies condition (3.3), since then $\alpha > (1 - \beta^{1/2})^{-2}$. This means that contrary to conclusions reached in [1, 2], in this case a surface charge can be produced not only in a retarding field $(E_1 < 0)$ but, also, when the ions are accelerated by the field $(E_1 > 0)$. The integral curve II in Fig. 6 relates to a positive field ahead of the wave front. In the case of the model of particles in the form of elastic solid spheres $a_t = 0.61$. If $\gamma = 1.4$, (3.3) is valid for $M_1 > 2.94$, while $E_m > 0$ when $M_1 > 4.65$.

Note that when the initial field intensity $E_1 < -\beta$, the allowance for thermal diffusion and fulfilment of (3.3) results in the generation of an additional surface charge which is higher than that generated by friction between ions and the neutral component taken into account in [2].

4. Shock wave structure with allowance in Ohm's law for the pressure, gradient, the convection term, and thermal diffusion. The analysis of integral curve behavior of Eqs. (1.14) and (1.15) in Sect. 2 has shown that, when $a_v = a_t = 0$, the allowance in Ohm's law for the convection term and pressure gradient does not provide new relationships at the shock wave frontdifferentfrom those that obtain when Ohm's law is used in the form (1.1) ($\mu = 0$), except for the following singularity.

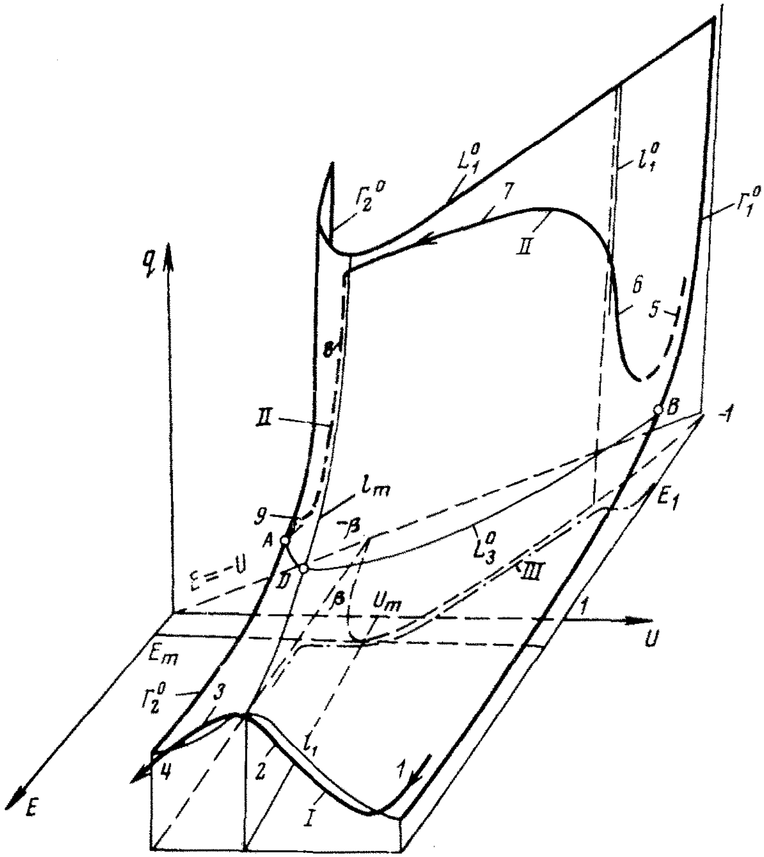

Fig.7

Within a certain range of conditions ahead of the wave front it is not possible to obtain a continuous structure with the use of the considered model, but it is possible to construct a solution by introducing at some section inside the structure a jump of volume charge density and maintaining all remaining parameters constant. A similar situation takes place when $\mu \neq 0$, $a_t \neq 0$, and $a_v = 0$.

The surface $L_1{}^\circ$ in whose δ-neighborhood pass integral curves which define the wave structure is shown in Fig. 7 in the region $\beta < u < 1$. The equation of surface $L_1{}^\circ$ is in this case of the form

$$q = [E + u + \alpha (u - 1) (u - \beta)]^{-1} \tag{4.1}$$

When $E_1 > E_m$, line l_1° has a maximum at point $u = u_m$, and when $E_1 \leqslant E_m$ it has asymptotes formed by generatrices of the parabolic cylinder

$$E = -u - \alpha (u - 1)(u - \beta) \tag{4.2}$$

Values of u_m and E_m are determined by formulas (3.2). Line l_m is the intersection line of surface L_1° and the plane $u = u_m$.

The integral curve I which passes near the initial point $(1, E_1, q_1)$ in the region where $L_3 < 0$ when $E_1 > E_m$, runs above surface L_1° in the μ-neighborhood of line Γ_1° (section 1), then deviates from the plane $u = 1$, intersects surface L_1°, and runs under it in its δ-neighborhood close to the plane $E = E_1$ (section 2). At point (u_m, E_1, q_m) where q_m is determined by (4.1) with $u = u_m$ and $E = E_1$ the curve intersects surface L_1° and then runs above it in the direction of decreasing u (section 3). Near the plane $u = \beta$ the curve turns and, without intersecting L_1°, runs along line Γ_2° (section 4). The described pattern of behavior of integral curve I may be established by the method set forth in Sect. 2. That integral curve defines the shock wave structure ahead of which the field is continuous $(E_2 = E_1)$. It is evident that if conditions ahead of the wave front satisfy inequality (3.3) inside the structure the volume charge density variation is non-monotonic. The maximum of q is at that point $\zeta = \zeta_m$ where the gas velocity $u = u_m$.

The integral curve II which passes through the small neighborhood of the initial point $(1, E_1, q_1)$ defines the shock wave structure at whose front the electric field is constant and $E < E_m$. Figure 7 shows the case when sections 5 - 9 of integral curve II lie in the region where $L_3 > 0$, which is possible when $E_1 < E_B$ and $E_m < E_A$ (parameters E_A and E_B are defined by formulas (2.1)).

Curve II runs near Γ_1° under surface L_1° (section 5), then, deviates from the plane $u = 1$ in the neighborhood of the initial point and, after intersecting surface L_1° runs above it along line l_1° close to the plane $E = E_1$ (section 6). Since l_1° has a vertical asymptote, the charge density q increases along curve II in the direction of decreasing u. With increasing q the integral curve gradually deviates from the plane $E = E_1$ and, when $q \sim \varepsilon^{-1}$, begins to proceed along the parabolic cylinder (4.2) in its δ-neighborhood in the direction of increasing E (section 7) and tends to approach the plane $u = u_m$. Then curve II intersects surface L_1° at a point of line l_m. After that intersection the derivative dq / ds changes its sign, and the charge density begins to decrease along the integral curve in the direction of decreasing u. Since the sign on derivative dE / ds remains unchanged, the integral curve runs under surface L_1° along the generatrix of the parabolic cylinder (4.2), which lies near the intersection line of planes $u = u_m$ and $E = E_m$ (section 8). At that stage curve II leaves the δ-neighborhood of surface L_1°, but remains in some of its δ^r-neighborhood ($0 < r < 1$). It can be seen from (1.16), (1.17), and (2.2) that, when $|L_1| \sim \delta^r$ and $q \sim \varepsilon^{-1}$, parameters α_1 and α_2 are of order δ^r, and $\alpha_3 = O(1)$. Consequently, the integral curve in section 8 is close to vertical, and the charge density there decreases rapidly, while the variation of parameters u and E is small. When parameter q reaches the order of unity, the integral curve enters the δ-neighborhood of surface L_1° and runs near the plane $E = E_m$ approaching the plane $u = \beta$, since there the derivatives du/ds and dq/ds are of the same order and the derivative dE/ds is small. Near the plane $u = \beta$ curve II turns in the direction of increasing E and runs along the line Γ_2° (section 9).

Interpretation of the various sections of integral curve II in the physical plane is similar to that given in Sects. 2 and 3. Sections 5 and 9 correspond to perfect subsonic flows, and sections 6 - 8 define the variations of flow parameters within the structure. The projection of curve II on the plane $q = 0$ (the dash-dot line III) shows that the field intensity behind the wave front is $E_2 = E_m$.

We denote by (u_m, E_D, q_D) the coordinates of point D of intersection of line l_m with line AB of the intersection of surfaces L_1° and L_3°. The values of E_D and q_D are determined by the equations $L_1 = 0$ and $L_3 = 0$ with $u = u_m$. If $E_B < E_1 < E_D$, the line l_1° of intersection of the plane $E = E_1$ with surface L_1° can have either one or two points of intersection with line AB. When $E_1 < E_A$ that point is in region $u > u_m$ in the first case, while in the second (when $E_A < E_1 < E_D$) the points lie on both sides of the plane $u = u_m$. In both cases $q = O(1)$ at the intersection points. These intersection points are singular points of Eq. (1.16) in the plane $E = \text{const}$. Analysis shows that the point lying in region $u > u_m$ is a focus (point F in Fig. 8), and the point lying in region $u < u_m$ is a saddle (point C in Fig. 8). A qualitative pattern of integral curve behavior is shown in Fig. 8 in their projection on the plane $E = E_1$ when $E_1 > E_m$.

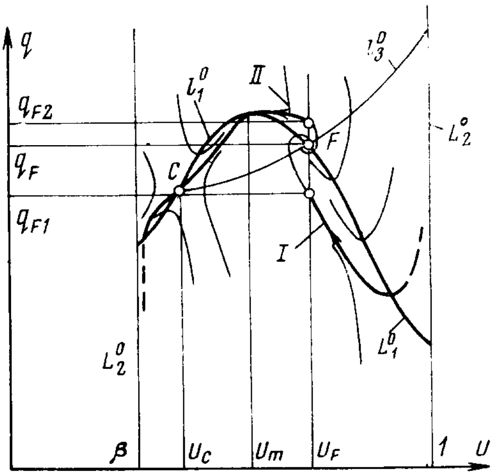

Fig. 8

The solution which determines the shock wave structure can be obtained using the two separatrices: curve I which curls around point F, and the integral curve II which passes through point C. As shown in Section 2, in order to pass from curve I to curve II when $u = u_F$, it is necessary to introduce a charge density jump and maintain all other parameters constant. Such jump can be smoothed out by taking into account one more dissipation mechanism, viz. the charged particle viscosity. Intensity of the shock with $\delta \ll 1$ is low.

5. **Shock wave structure when $a_t \neq 0, a_v \neq 0$, and $\mu = 0$.** The simultaneous introduction in Ohm's law of terms related to thermal diffusion and viscous momentum transport of the mixture in the electric field also yields under specific conditions a new relationship for flow parameters at the wave front. For simplicity let us consider the case in which $\mu = 0$. The input system of equations reduces to the equation

$$dE/du = \frac{\varepsilon}{L_2 L_5}, \quad L_5 = \frac{1}{u} E g_1 + g_2, \quad g_1 = u + c(u-1)(u-\beta) \tag{5.1}$$
$$g_2 = u + \alpha(u-1)(u-\beta), \quad c = a_v(\gamma+1)/4\gamma.$$

The behavior of integral curves of Eq. (5.1) considerably depends on the position of curve L_5° whose equation in region $\beta < u < 1$ in the uE-plane is $L_5 = 0$. That curve passes the intersection points of straight lines $u = \beta$ and $u = 1$ with line $E = -u$, and is the indicatrix of the vertical directions of integral curves of Eq. (5.1).

Let us, first, consider the case when the equation $g_1 = 0$ has no roots in the interval $\beta < u < 1$. The constraint on flow parameters that corresponds to that condition is of the form $c < (1 - \beta^{1/2})^{-2}$. Let also $\alpha > c > 0$ and let the inequality (3.3) for which the maximum (3.2) of parabola $E = -g_2$ lies in region $\beta < u < 1$, be satisfied. The values of E determined by the equality $E = ug_1^{-1}$ in the interval $\beta < u < 1$ are greater than unity. Hence in region $E > 0$ curve L_5° lies in the interval $0 < u < 1$ above parabola $E = -g_2$, and below the latter in region $E < 0$. In Fig. 3 the dash line represents L_5° and the dash-dot lines 6 and 7 represent, respectively, lines $E = ug_1^{-1}$ and $E = -g_2$ in the case when point (3.2) lies in region $E < 0$.

Analysis of the behavior of integral curves of Eq. (5.1) is similar to that carried out in Sect. 3. The position of maxima of curve L_5° in the interval $\beta < u < 1$ is significant. The coordinates of these maxima are determined by the equations

$$c(\beta - u^2) g_2 + ug_1 (1 + 2\alpha u - \alpha - \alpha\beta) = 0 \qquad (5.2)$$
$$E = -ug_2 g_1^{-1}$$

With the indicated above constraints on parameters and with the inequality $\alpha - c > (1 - \beta)^{-1}$ satisfied, there exists in the interval $\beta < u < 1$ at least one out of two possible maxima in the general case. We denote by (u_*, E_*) the coordinates of the highest maximum of curve L_5° in the interval $\beta < u < 1$.

Analysis of the structure shows that when the field intensity ahead of the wave front $E_1 \geqslant E_*$ and $E_* \geqslant -\beta$, the electric field at the wave front is continuous $(E_2 = E_1)$, and a surface charge is absent.

If the field intensity ahead of the wave from $E_1 < E_*$, the electric field at the wave front is discontinuous, and a surface charge is formed in the electric field. The charge is produced by friction of charged particles with neutral ones, thermal diffusion, and by the effect of viscous momentum transfer of the mixture on diffusion. If in this case $E_* \geqslant -\beta$, the field intensity behind the wave front satisfies the relation

$$E_2 = E_* \qquad (5.3)$$

If $E_* < -\beta$, the field behind the wave front is $E_2 = -\beta$. In that case thermal diffusion and viscous momentum transfer of the mixture do not contribute to the formation of surface charge. Note that when $\alpha = (1 - \beta^{1/2})^{-2}$, then $u_* = \beta^{1/2}$ and $E_* = 0$, hence the field behind the the front is $E_2 = 0$.

Let us now consider the case when the parameters ahead of the front are such that $c > (1 - \beta^{1/2})^{-2}$. The equation $g_1 = 0$ has then roots in the interval $\beta < u < 1$ and, consequently, line L_5° has vertical asymptotes in that interval.

Let $\alpha < c$. Curve L_5° has three branches (fig. 9). We denote by (u_{**}, E_{**}) the coordinates of the central branch minimum whose ordinate is the smallest. The abscissa u_{**} of that minimum lies between the roots of equation

$g_1 = 0$ and is the root of the first of Eqs. (5.2). The left-and right-hand branches of $L_5{}^\circ$ lie in the interval $\beta < u < 1$ below the straight line $E = -u$.

Figure 9 shows that, when the electric field ahead of the wave front $E_1 < -\beta$, a surface charge is generated at the wave and the field behind the front satisfies the relation $E_2 = -\beta$. The structure of shock waves with discontinuities is defined by integral curves of type I. Note that the surface charge is the same as in the case when only friction between charged and neutral particles is taken into account, but the pattern of flow parameter variation within the structure is in the considered case different.

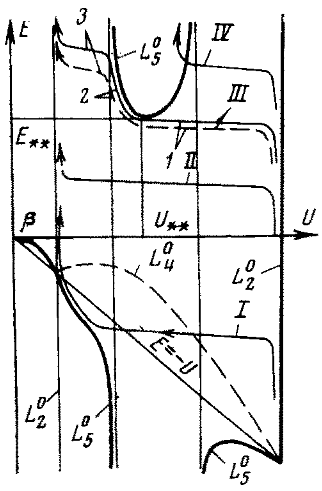

Fig. 9

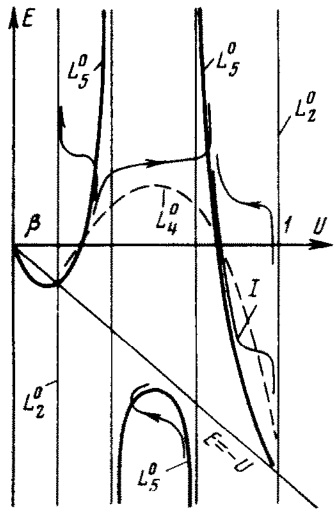

Fig. 10

Analysis of the structure shows that, when $-\beta < E_1 < E_{**}$, the electric field at the wave front is continuous and there is no surface charge (integral curve of type II).

Let us consider integral curves of type III which pass through the neighborhood of the initial point $(1, E_1)$ and

$$E_1 = E_{**} \tag{5.4}$$

Each of these integral curves has a nearly horizontal section 1 which deviates from the supersonic branch of line $L_2{}^\circ$ in the neighborhood of point $(1, E_1)$ to which in the physical plane corresponds to the region of abrupt velocity change in an almost constant field intensity. Section 2 which runs along the central branch of line $L_5{}^\circ$ in its small neighborhood defines in the physical plane a narrow region of abrupt change of velocity and electric field. Then these integral curves deviate from line $L_5{}^\circ$ at various electric field intensities bounded by the condition

$$E \geqslant E_{**} \tag{5.5}$$

and run with a small slope in the direction of the subsonic branch of line $L_2{}^\circ$ (section 3). These integral curves define the shock wave structure with a jump of the electric field at the wave front. The parameters of flow ahead of such waves are linked by the supplementary relation (5.4). To determine the state behind the front it is necessary to specify one of the flow parameters.

The electric field intensity may be specified within the limits (5.5) Electrohydrodynamic waves of a similar kind but related to a different physical situation were considered in [2]. When the initial field intensity $E_1 > E_{**}$, shock waves defined by the model considered here have no structure. This is shown by the behavior of integral curves of type IV which depart from line $u = 1$ when $E_1 > E_{**}$, intersect the central branch of line L_5°, enter the region of negative electric current, and then move in that region along line L_5° with a finite positive slope.

Let $\alpha > c$. Curve L_5° has in region $\beta < u < 1$ three branches of which the central one lies in region $E < 0$, and the left-and right-hand branches approach the vertical asymptotes with positive and negative slopes, respectively. In that case shock waves have no structure. This is so because none of the integral curves departing from the supersonic branch of line L_2° can intersect the right-hand branch of line L_5°, since throughout the region to the right of the right-hand asymptote the slope of integral curves approaching sections of the right-hand branch of line L_5° and the slopes of those sections are negative, while an intersection is possible only with a line of vertical slope (Fig. 10). Let us briefly consider the case of $c = (1 - \beta^{1/2})^{-2}$. The equation $g_1 = 0$ has then a multiple root, hence line L_5° has in the interval $\beta < u < 1$ two branches separated by the vertical asymptote $u = \beta^{1/2}$. When $\alpha < c$, both branches are in region $E < 0$ and $\beta < u < 1$ below the straight line $E = -u$. The results of structure investigation are the same as when $\alpha = c = 0$. When $\alpha > c$, the shock waves have no structure, since both branches tend to the asymptote in the region of positive E. This case is similar to that of $\alpha > c > (1 - \beta^{1/2})^{-2}$.

If $\alpha < 0$ and $c < 0$, then for $|\alpha| < |c|$ the results are similar to those when parameters α and c are positive and equations $g_1 = 0$ and $g_2 = 0$ have no roots in the interval $\beta < u < 1$. When $|\alpha| > |c|$ the supplementary relationships are the same as when $\alpha = c = 0$.

REFERENCES

1. Gogosov, V. V. and Polianskii, V. A., Discontinuities in electrohydrodynamics. PMM. Vol. 35, No. 5, 1971.
2. Gogosov, V. V. and Polianskii, V. A., The structure of electrohydrodynamic shock waves. PMM, Vol. 36, No. 5, 1972.
3. Gogosov, V. V., Polianskii, V. A., Semenova, I. P. and Iakubenko, A. E., Equations of electrohydrodynamics and transport coefficients in a strong electric field. Izv. Akad. Nauk SSSR, MZhG, No. 2, 1969.
4. Hirschfelder, J., Curtiss, Ch. and Bird, R., Molecular Theory of Gases and Liquids. Moscow, Izd. Inostr. Lit., 1961.
5. Macdaniel, I. and Mason, E., Mobility and Diffusion of Ions in Gases. Moscow, "Mir", 1976
6. Gogosov, V. V., Korovin, V. M., Polianskii, V. A. and Shaposhnikova, G. A., Isothermic shocks in electrohydrodynamics. PMM, Vol. 40, No. 3, 1976.
7. Gogosov, V. V. and Polianskii, V. A., Electrohydrodynamics: Problems and applications, fundamental equations, discontinuous solutions. Attainments of science and technology. Mekhanika Zhidkosti i Gaza, Vol. 10, Moscow, VINITI, 1976.
8. Mises, R., Mathematical Theory of Compressible Fluid flow. Moscow, Izd. Inostr. Lit., 1961.
9. Rashevskii, P. K., Course of Differential Geometry. Moscow, Gostekhizdat, 1956.

Translated by J. J. D.